On Statistical Regularity

Before one proceeds to the notion of a probability measure defined on a sample space one needs to comprehend the notion of what a measurable space is and formally define what an event in such a space is. Towards this purpose consider a random exprerimment with an associated sample space Ω and let us specifically look at the collection of subsets of Ω. Let A and B be two such subsets in the collection. The collection of subsets of Ω forms a field $\mathcal{F}$ if the collection is closed under set union, intersection and complement operations, i.e, if following relations are true:

1. $\phi \in \mathcal{F}, \quad \Omega \in \mathcal{F}$.
2. $A \cup B \in \mathcal{F}, \quad A \cap B \in \mathcal{F}$.
3. $A^c \in \mathcal{F}, \quad B^c \in \mathcal{F}$.

Let us now look at a sequence of subsets in Ω, $E_i,\ i = 1, 2, \ldots$ that form a field $\mathcal{F}$. The field $\mathcal{F}$ further can be further classified as a σ-field when it is closed under any countable set of unions and intersections, i.e.,

$$E_i \in \mathcal{F} \longleftrightarrow \bigcup_{i=1}^{\infty} E_i \in \mathcal{F}, \quad \bigcap_{i=1}^{\infty} E_i \in \mathcal{F}.$$

Suffice it to say that when we refer to an event in this course we will refer to only those statistically regular sets in the field $\mathcal{F}$ that can be expressed as a countable union or intersection of the elements of the the field of subsets, i.e.,

$$A = \bigcap_{i=1}^{\infty} E_i, \quad B = \bigcup_{i=1}^{\infty} E_i.$$

In general not all subsets of Ω can be assigned a probability consistent with the axioms of probability. For example, in the context of uncountable sets Ω such as the real-line, not all subsets in Ω may satisfy the axioms of probability and only a smaller subset of these elements in the field of subsets $\mathcal{F}$ can be assigned a probability measure. This smaller subset of $\mathcal{F}$ will be refered to as the Borel-field of events. The space of events defined by the pair $(\mathcal{F}, \Omega)$ is called a measurable space, when $\mathcal{F}$ is a σ-field. The probability measure P that associates with elements in the σ-field $\mathcal{F}$ a corresponding real number in the interval $[0, 1]$ can then be defined via the axioms of probability. The triplet defined via $(\Omega, \mathcal{F}, P)$ is called a probability space.